

## Bayesian Complex Amplitude Estimation and Adaptive Matched Filter Detection in Low-Rank Interference

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**Abstract**—We propose a Bayesian method for complex amplitude estimation in low-rank interference. We assume that the received signal follows the generalized multivariate analysis of variance (GMANOVA) patterned-mean structure and is corrupted by low-rank spatially correlated interference and white noise. An iterated conditional modes (ICM) algorithm is developed for estimating the unknown complex signal amplitudes and interference and noise parameters. We also discuss initialization of the ICM algorithm and propose a (non-Bayesian) adaptive-matched-filter (AMF) signal detector that utilizes the ICM estimation results. Numerical simulations demonstrate the performance of the proposed methods.

**Index Terms**—Adaptive matched filter (AMF), Bayesian analysis, complex amplitude estimation, iterated conditional (ICM) modes, low-rank interference.

### I. INTRODUCTION

The topic of signal detection and estimation in low-rank interference has recently attracted considerable attention (see, e.g., [1]–[6] and references therein) as it has great potential in many signal processing applications facing the curse-of-dimensionality problem and data (or “snapshot”) constraints [4], [5]. In space–time adaptive processing (STAP) for radar, low-rank interference is due to clutter and jamming (see, e.g., [6]–[8]). In [3] and [9], maximum-likelihood (ML) and least-squares estimators of low-rank covariance matrices are derived assuming that secondary (interference-plus-noise only) data is available. Structured ML covariance estimation from interference-only measurements is discussed in [10]. Intrinsic Cramér–Rao bounds (CRBs) for low-rank subspace estimation are developed in [1] and low-rank subspace tracking is discussed in [11]. Bayesian and non-Bayesian approaches for complex amplitude estimation have been developed and analyzed in [12], [13] and [14]–[17] (see also references therein) assuming unstructured covariance matrix of interference and noise. A Bayesian approach for estimating interference-plus-noise covariance matrices in knowledge-aided radar is outlined in [18], where numerous examples of available *a priori* information are given for the radar problem. The methods in [12]–[18] ignore the low-rank structure of the interference. In this correspondence, we develop an iterated conditional modes (ICM) algorithm for Bayesian estimation of complex signal amplitudes in low-rank interference and propose a (non-Bayesian) adaptive-matched-filter (AMF) detector that utilizes the ICM estimates of the unknown parameters. The computational complexity of (each step of) our iterative scheme increases only *linearly* with the array size and is therefore determined by the ranks of signal and interference than by the array size.

Measurement model and prior specifications are introduced in Section II, where we also justify use of the Bayesian methodology. In Section III, we develop the ICM algorithm for Bayesian com-

plex amplitude estimation, discuss its initialization and interference rank estimation (Section III-A), and derive an AMF signal detector (Section III-B). In Section IV, we evaluate the performance of the proposed methods via numerical simulations. Concluding remarks are given in Section V.

### II. MEASUREMENT MODEL AND PRIOR SPECIFICATIONS

Denote by  $\mathbf{y}(t)$  an  $m \times 1$  complex data vector (snapshot) received at time  $t$  and assume that we have collected  $N$  snapshots. Consider the following model for the received snapshots:

$$\mathbf{y}(t) = AX\boldsymbol{\phi}(t) + B\mathbf{h}(t) + \mathbf{e}(t), \quad t = 1, \dots, N \quad (2.1)$$

where the first term describes the generalized multivariate analysis of variance (GMANOVA) patterned mean-signal structure with a known  $m \times q$  signal array–response matrix  $A$  ( $m \geq q$ ), known  $d \times 1$  signal temporal response vectors  $\boldsymbol{\phi}(t)$ ,  $t = 1, 2, \dots, N$  ( $d \leq N$ ), and an  $q \times d$  matrix  $X$  of unknown complex signal amplitude coefficients. (See [14] and references therein for a detailed exposition on the GMANOVA model and its applications.) The second term in (2.1) corresponds to low-rank interference described by the following:

- an unknown  $m \times r$  interference array–response matrix  $B$  ( $m \geq r$ );
- random zero-mean independent, identically distributed (i.i.d.)  $r \times 1$  vectors  $\mathbf{h}(t)$ ,  $t = 1, 2, \dots, N$  of interference signals following a circularly symmetric complex Gaussian distribution with unknown covariance  $\sigma^2 \Sigma$ . (Here,  $\Sigma$  is a normalized covariance matrix of the interference signals.)

Finally,  $\mathbf{e}(t)$  is zero-mean circularly symmetric i.i.d. complex Gaussian noise with covariance  $\sigma^2 I_m$ , where  $\sigma^2$  is unknown noise variance and  $I_m$  denotes the identity matrix of size  $m$ . Denote by  $\boldsymbol{\xi}$  the set of all unknown parameters:

$$\boldsymbol{\xi} = \{X, B, \Sigma, \sigma^2\} \quad (2.2)$$

where  $\Sigma$  is a positive-semidefinite Hermitian matrix. We allow the interference rank  $r$  to be *unknown* as well, estimated in the initialization stage (i.e., separately from the estimation of  $\boldsymbol{\xi}$ ); see Section III-A.

We define the data, temporal signal response, and interference-signal matrices, as follows:

$$\begin{aligned} Y &= [\mathbf{y}(1) \cdots \mathbf{y}(N)] \\ \Phi &= [\boldsymbol{\phi}(1), \boldsymbol{\phi}(2) \cdots \boldsymbol{\phi}(N)] \\ H &= [\mathbf{h}(1), \mathbf{h}(2) \cdots \mathbf{h}(N)] \end{aligned} \quad (2.3)$$

and assume that  $\Phi$  has full rank equal to  $d$ . Before we proceed, let us introduce the following definition: a matrix-variate circularly symmetric complex Gaussian probability density function (pdf) of an  $p \times q$  random matrix  $Z$  with mean  $M$  (of size  $p \times q$ ) and positive-definite covariance matrices  $S$  and  $\Phi$  (of dimensions  $p \times p$  and  $q \times q$ , respectively) is

$$\begin{aligned} \mathcal{N}_{p \times q}(Z; M, S, \Phi) &= \frac{1}{\pi^{pq} |S|^q \cdot |\Phi|^p} \\ &\cdot \exp \left\{ -\text{tr} \left[ \Phi^{-1} (Z - M)^H S^{-1} (Z - M) \right] \right\} \end{aligned} \quad (2.4)$$

where  $|\cdot|$ ,  $\text{tr}(\cdot)$ , and “ $H$ ” denote the determinant, trace, and Hermitian (conjugate) transpose. This definition is an extension of [19, eq. (2.7.4)] and [20, Definition 1.1] to the complex-data case.

The model description (2.1)–(2.3) leads to the likelihood function of the unknown parameters

$$p(Y|\boldsymbol{\xi}) = \mathcal{N}_{m \times N}(Y; AX\Phi, \mathcal{R}(B, \Sigma, \sigma^2), I_N) \quad (2.5a)$$

Manuscript received December 16, 2005; revised May 9, 2006. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Fulvio Gini. This work was supported by the National Science Foundation under Grant CCF-0545571.

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Digital Object Identifier 10.1109/TSP.2006.887151

where  $p(Y|\xi)$  denotes the conditional pdf of  $Y$  given  $\xi$ ,

$$\begin{aligned} \mathcal{R}(B, \Sigma, \sigma^2) &= \mathbb{E} \left\{ [B\mathbf{h}(t) + \mathbf{e}(t)] [B\mathbf{h}(t) + \mathbf{e}(t)]^H \right\} \\ &= \sigma^2 R(B, \Sigma) \end{aligned} \quad (2.5b)$$

is the spatial covariance matrix of interference and noise, and

$$R(B, \Sigma) = I_m + B\Sigma B^H. \quad (2.5c)$$

This measurement model is equivalently represented using a two-stage hierarchical formulation:

$$p(Y|X, B, H, \sigma^2) = \mathcal{N}_{m \times N}(Y; AX\Phi + BH, \sigma^2 I_m, I_N) \quad (2.6a)$$

$$p(H|\Sigma, \sigma^2) = \mathcal{N}_{r \times N}(H; 0_{r \times N}, \sigma^2 \Sigma, I_N) \quad (2.6b)$$

where  $0_{r \times N}$  denotes the  $r \times N$  matrix of zeroes.

#### A. Prior Specifications

We assume that the unknown parameters are independent *a priori*, i.e.,

$$\pi(\xi) = \pi(X, B, \Sigma, \sigma^2) = \pi(X) \cdot \pi(B) \cdot \pi(\Sigma) \cdot \pi(\sigma^2). \quad (2.7a)$$

Here,  $\pi(\xi)$  denotes the prior pdf of  $\xi$  and analogous notation is used for the prior pdf's of the components of  $\xi$ . Let us adopt an (improper) "flat" Jeffrey's noninformative prior pdf for the complex signal amplitudes<sup>1</sup>

$$\pi(X) \propto 1 \quad (2.7b)$$

and select *conjugate prior pdf's* for the noise and interference parameters<sup>2</sup>:

$$\begin{aligned} \pi(\sigma^2) &= \text{Inv-}\chi^2(\sigma^2; 2\nu_{\sigma^2}, \sigma_0^2) \\ &\propto (\sigma^2)^{-(1+\nu_{\sigma^2})} \cdot \exp(-\nu_{\sigma^2} \cdot \sigma_0^2 / \sigma^2) \end{aligned} \quad (2.7c)$$

$$\pi(B) = \mathcal{N}_{m \times r}(B; M_B, I_m, \Gamma_B) \quad (2.7d)$$

$$\begin{aligned} \pi(\Sigma) &= \text{Inv-Wishart}_r(\Sigma; \nu_\Sigma, [(\nu_\Sigma - r)A_0]^{-1}) \\ &\propto |\Sigma|^{-(r+\nu_\Sigma)} \cdot \exp[-(\nu_\Sigma - r) \cdot \text{tr}(\Sigma^{-1}A_0)] \end{aligned} \quad (2.7e)$$

where we have the following:

- $\text{Inv-}\chi^2(\sigma^2; 2\nu_{\sigma^2}, \sigma_0^2)$  denotes the pdf of a scaled inverse chi-square distribution with  $2\nu_{\sigma^2}$  degrees of freedom and a scale parameter  $\sigma_0^2$ , see [21, p. 50 and App. A];<sup>3</sup>
- $\text{Inv-Wishart}_r(\Sigma; \nu_\Sigma, \Sigma_0^{-1})$  denotes the pdf of an  $r \times r$  random matrix  $\Sigma$  having complex inverse Wishart distribution with  $\nu_\Sigma$  degrees of freedom and positive definite Hermitian  $r \times r$  scale matrix  $\Sigma_0$ ;<sup>4</sup>
- prior mean and precision matrices  $M_B$  and  $\Gamma_B^{-1}$  (of dimensions  $m \times r$  and  $r \times r$ , respectively) quantify our prior knowledge about the interference array response  $B$ .

The degrees of freedom  $\nu_\Sigma$  and  $\nu_{\sigma^2}$  can be interpreted as numbers of virtual "observations" describing our prior knowledge about  $\Sigma$  and  $\sigma^2$ , where each "observation" is equal to  $A_0$  or  $\sigma_0^2$ , respectively. Prior information about the noise level  $\sigma^2$  is available in many applications (e.g., in systems operating at the microwave frequencies, see [3]), thus justifying use of informative priors for  $\pi(\sigma^2)$  and the Bayesian approach in general. Complex inverse-Wishart priors have been utilized in [18] to improve estimation of interference-plus-noise covariance matrices in knowledge-aided radar. The interference array response prior (2.7d)

<sup>1</sup>See [21, Ch. 2.9] for an introduction to noninformative prior distributions.

<sup>2</sup>Utilizing conjugate priors simplifies Bayesian computations.

<sup>3</sup>Provided that  $\nu_{\sigma^2} > 1$ , the mean of the  $\text{Inv} - \chi^2(\sigma^2; 2\nu_{\sigma^2}, \sigma_0^2)$  pdf is  $\nu_{\sigma^2} / (\nu_{\sigma^2} - 1) \cdot \sigma_0^2$ .

<sup>4</sup>Provided that  $\nu_\Sigma > r$ , the mean of the  $\text{Inv} - \text{Wishart}_r(\Sigma; \nu_\Sigma, \Sigma_0^{-1})$  pdf is  $(\nu_\Sigma - r)^{-1} \cdot \Sigma_0$  (see [22, eq. (39)]); consequently, the mean of the prior pdf in (2.7e) is  $A_0$ . See [22] for a detailed discussion on complex inverse-Wishart distribution.

can be constructed from previous estimates of  $B$ . The Bayesian framework allows us to utilize sequential-Bayesian ideas and *track* the interference. In practice, we may select the precision  $\Gamma_B^{-1}$  to be a diagonal matrix whose diagonal elements describe prior precisions of the interference array-response vectors (i.e., the columns of  $B$ ). Lack of prior information on  $B$  can be expressed by choosing a "flat" noninformative prior with  $\Gamma_B^{-1} = 0_{r \times r}$ .

### III. BAYESIAN ESTIMATION OF COMPLEX SIGNAL AMPLITUDES AND APPLICATION TO ADAPTIVE DETECTION

We first develop a Bayesian approach for estimating the unknown parameters  $\xi$  under the measurement and prior models in Section II and then discuss its application to adaptive signal detection. The joint posterior pdf of the parameters  $\xi$  and random interference signals  $H$  is given by

$$\begin{aligned} p(\xi, H|Y) &\propto p(Y|X, B, H, \sigma^2) \cdot p(H|\Sigma, \sigma^2) \cdot \pi(X) \cdot \pi(B) \\ &\quad \cdot \pi(\Sigma) \cdot \pi(\sigma^2) \\ &\propto |\sigma^2 I_m|^{-N} \exp \left\{ -\text{tr}[(Y - AX\Phi - BH) \right. \\ &\quad \quad \left. \times (Y - AX\Phi - BH)^H] / \sigma^2 \right\} \\ &\quad \cdot |\sigma^2 \Sigma|^{-N} \cdot \exp \left[ -\text{tr}(\Sigma^{-1} H H^H) / \sigma^2 \right] \\ &\quad \cdot \exp \left\{ -\text{tr} \left[ (B - M_B) \Gamma_B^{-1} (B - M_B)^H \right] \right\} \\ &\quad \cdot |\Sigma|^{-(r+\nu_\Sigma)} \cdot \exp \left[ -(\nu_\Sigma - r) \cdot \text{tr}(\Sigma^{-1} A_0) \right] \\ &\quad \cdot (\sigma^2)^{-(1+\nu_{\sigma^2})} \cdot \exp \left( -\nu_{\sigma^2} \cdot \sigma_0^2 / \sigma^2 \right) \end{aligned} \quad (3.1a)$$

implying that

$$\begin{aligned} p(H|\xi, Y) &= \mathcal{N}_{r \times N} \left( H; (\Sigma B^H B + I_r)^{-1} \Sigma B^H (Y - AX\Phi), \right. \\ &\quad \left. \sigma^2 \cdot (\Sigma B^H B + I_r)^{-1} \Sigma, I_N \right) \end{aligned} \quad (3.1b)$$

$$\begin{aligned} p(B|X, H, \Sigma, \sigma^2, Y) &= \mathcal{N}_{m \times r} \left( B; \left[ (Y - AX\Phi) H^H + \sigma^2 M_B \Gamma_B^{-1} \right] \right. \\ &\quad \cdot \left( H H^H + \sigma^2 \Gamma_B^{-1} \right)^{-1}, I_m, \\ &\quad \left. \sigma^2 \cdot \left( H H^H + \sigma^2 \Gamma_B^{-1} \right)^{-1} \right) \end{aligned} \quad (3.1c)$$

which follow by keeping the terms in (3.1a) that depend on  $H$  and  $B$  (respectively) and completing the squares in the exponents of the resulting expressions. The result (3.1b) is consistent with the well-known result in [23, eqs. (15.64)–(15.67)]. Integrating  $H$  out from (3.1a) yields the posterior pdf of  $\xi$

$$\begin{aligned} p(\xi|Y) &\propto p(Y|\xi) \cdot \pi(X) \cdot \pi(B) \cdot \pi(\Sigma) \cdot \pi(\sigma^2) \\ &\propto \frac{1}{|\sigma^2 R(B, \Sigma)|^N} \\ &\quad \cdot \exp \left\{ -\text{tr} \left[ R(B, \Sigma)^{-1} (Y - AX\Phi) (Y - AX\Phi)^H \right] / \sigma^2 \right\} \\ &\quad \cdot \exp \left\{ -\text{tr} \left[ (B - M_B) \Gamma_B^{-1} (B - M_B)^H \right] \right\} \\ &\quad \cdot |\Sigma|^{-(r+\nu_\Sigma)} \cdot \exp \left[ -(\nu_\Sigma - r) \cdot \text{tr}(\Sigma^{-1} A_0) \right] \\ &\quad \cdot (\sigma^2)^{-(1+\nu_{\sigma^2})} \cdot \exp \left( -\nu_{\sigma^2} \cdot \sigma_0^2 / \sigma^2 \right) \end{aligned} \quad (3.2)$$

and the conditional pdf  $p(X|B, \Sigma, \sigma^2, Y)$  follows by keeping the terms in (3.2) that depend on  $X$  and completing the squares in the exponent:

$$\begin{aligned} p(X|\sigma^2, B, \Sigma, Y) &= \mathcal{N}_{q \times d} \left( X; \hat{X}(B, \Sigma), \right. \\ &\quad \left. [A^H \mathcal{R}(B, \Sigma, \sigma^2)^{-1} A]^{-1}, (\Phi \Phi^H)^{-1} \right) \end{aligned} \quad (3.3a)$$

where

$$\hat{X}(B, \Sigma) = [A^H R(B, \Sigma)^{-1} A]^{-1} \times A^H R(B, \Sigma)^{-1} Y \Phi^H (\Phi \Phi^H)^{-1} \quad (3.3b)$$

which is consistent with the real-data result in [20, Theorem 6.7]. If we “vectorize”  $X$ , then (3.3a) implies that the posterior covariance matrix of  $\text{vec}(X)$  given  $\sigma^2$ ,  $B$ ,  $\Sigma$ , and  $Y$  is equal to the (non-Bayesian) CRB of  $\text{vec}(X)$  in [17, eq. (66)] and [15, App. 4.D].<sup>5</sup> Let us integrate  $X$  out from the joint pdf  $p(X, \sigma^2 | B, \Sigma, Y)$  as shown in (3.4) at the bottom of the page, see also (3.2) and (3.3a). Here,  $\hat{\sigma}^2(B, \Sigma)$  denotes the mode of the pdf in (3.4b) (see [21, App. A]) and (3.4b) follows by noting that  $p(\sigma^2, B, \Sigma, Y)$  does not depend on  $X$  and setting  $X = \hat{X}(B, \Sigma)$  in (3.4a).

We now estimate the unknown parameters  $\xi$  using an ICM approach [24, Sec. 4], [25, Ch. 10.2.1], [26, Ch. 6.2.2]. Our ICM algorithm consists of cycling between the following steps.

- 1) Fix  $B = \hat{B}$ ,  $\Sigma = \hat{\Sigma}$  and estimate  $X$  by maximizing the conditional posterior pdf  $p(X | B, \Sigma, \sigma^2, Y)$  [see also (3.3b)]:

$$\hat{X} = \hat{X}(B, \Sigma). \quad (3.5a)$$

- 2) Fix  $B = \hat{B}$ ,  $\Sigma = \hat{\Sigma}$  and estimate  $\sigma^2$  by maximizing the conditional posterior pdf  $p(\sigma^2 | B, \Sigma, Y)$  in (3.4):

$$\hat{\sigma}^2 = \hat{\sigma}^2(B, \Sigma). \quad (3.5b)$$

- 3) Fix  $B = \hat{B}$ ,  $X = \hat{X}$ ,  $\sigma^2 = \hat{\sigma}^2$  and estimate  $\Sigma$  by maximizing the conditional posterior pdf  $p(\Sigma | X, B, \sigma^2, Y)$ . For this purpose, we utilize an expectation-maximization (EM) step where we treat the random interference signals  $H$  as the *unobserved* (or missing) data:

$$\hat{\Sigma} = \arg \max_{\Sigma} E_{H|\Sigma=\hat{\Sigma}_p, X, B, \sigma^2, Y} \{\ln p(\xi, H | Y)\} \quad (3.5c)$$

where  $\Sigma_p$  denotes the estimate of  $\Sigma$  from the previous iteration and  $E_{H|\Sigma, X, B, \sigma^2, Y}$  the expectation with respect to  $p(H | \Sigma, X, B, \sigma^2, Y) = p(H | \xi, Y)$  in (3.1b). (See [21, Ch.

<sup>5</sup>Here, the  $\text{vec}$  operator stacks the columns of a matrix one below another into a single-column vector.

12.3] for a detailed exposition on EM-type algorithms in the Bayesian context.) Consequently

$$\hat{\Sigma} = \frac{1}{N + r + \nu_{\Sigma}} \cdot \left[ \hat{H} \hat{H}^H / \sigma^2 + N \cdot (\Sigma_p B^H B + I_r)^{-1} \Sigma_p + (\nu_{\Sigma} - r) \cdot A_0 \right] \quad (3.5d)$$

where  $\hat{H} = (\Sigma_p B^H B + I_r)^{-1} \Sigma_p B^H (Y - AX\Phi)$ .

- 4) Fix  $X = \hat{X}$ ,  $\Sigma = \hat{\Sigma}$ ,  $\sigma^2 = \hat{\sigma}^2$  and estimate  $B$  by maximizing the conditional posterior pdf  $p(B | X, \Sigma, \sigma^2, Y)$ . As before, we treat  $H$  as the missing data and utilize the following EM step:

$$\hat{B} = \arg \max_B E_{H|B=B_p, X, \Sigma, \sigma^2, Y} \{\ln p(\xi, H | Y)\} \quad (3.5e)$$

where  $B_p$  denotes the estimate of  $B$  from the previous iteration. Consequently

$$\hat{B} = \left\{ (Y - AX\Phi) \tilde{H}^H + \sigma^2 M_B \Gamma_B^{-1} \right\} \cdot \left[ \tilde{H} \tilde{H}^H + N \cdot \sigma^2 \cdot (\Sigma B_p^H B_p + I_r)^{-1} \Sigma + \sigma^2 \Gamma_B^{-1} \right]^{-1} \quad (3.5f)$$

where  $\tilde{H} = (\Sigma B_p^H B_p + I_r)^{-1} \Sigma B_p^H (Y - AX\Phi)$ .

This iterative scheme can also be viewed as a parameter-expanded expectation-maximization (PX-EM) algorithm [21, Ch. 12.3]. Here, *parameter expansion* refers to the fact that  $B$  and  $\Sigma$  overparameterize the interference. The computation of  $R(B, \Sigma)^{-1}$  in Steps 1) and 2) is performed efficiently using the matrix inversion lemma (e.g., [27, Theorem 18.2.8]):

$$R(B, \Sigma)^{-1} = (I_m + B \Sigma B^H)^{-1} = I_m - B (\Sigma B^H B + I_r)^{-1} \Sigma B^H \quad (3.6)$$

which requires inversion of an  $r \times r$  matrix and thus leads to significant computational savings compared with directly inverting  $R(B, \Sigma)$ . We utilize (3.6) to simplify Steps 1) and 2) and summarize the resulting ICM algorithm: compute

$$P^{(i)} = \left[ \Sigma^{(i)} \left( B^{(i)} \right)^H B^{(i)} + I_r \right]^{-1} \Sigma^{(i)} \quad (3.7a)$$

$$X^{(i)} = \left[ A^H A - A^H B^{(i)} P^{(i)} \left( B^{(i)} \right)^H A \right]^{-1} \cdot \left[ A^H Y - A^H B^{(i)} P^{(i)} \left( B^{(i)} \right)^H Y \right] \Phi^H (\Phi \Phi^H)^{-1} \quad (3.7b)$$

$$\begin{aligned} p(\sigma^2 | B, \Sigma, Y) &= \frac{p(X, \sigma^2 | B, \Sigma, Y)}{p(X | \sigma^2, B, \Sigma, Y)} \\ &\propto \frac{|\sigma^2 R(B, \Sigma)|^{-N} \cdot \exp(-\text{tr}[R(B, \Sigma)^{-1} (Y - AX\Phi)(Y - AX\Phi)^H] / \sigma^2) \cdot (\sigma^2)^{-(1+\nu_{\sigma^2})} \cdot \exp(-\nu_{\sigma^2} \cdot \sigma_0^2 / \sigma^2)}{(\sigma^2)^{-qd} \cdot \exp(-\text{tr}\left\{ \left[ X - \hat{X}(B, \Sigma) \right]^H A^H R(B, \Sigma)^{-1} A \left[ X - \hat{X}(B, \Sigma) \right] \Phi \Phi^H \right\} / \sigma^2)} \end{aligned} \quad (3.4a)$$

$$\begin{aligned} &\propto (\sigma^2)^{-(mN+1+\nu_{\sigma^2}-qd)} \cdot \exp(-\text{tr}\{R(B, \Sigma)^{-1} [Y - A\hat{X}(B, \Sigma)\Phi][Y - A\hat{X}(B, \Sigma)\Phi]^H\} / \sigma^2 - \nu_{\sigma^2} \cdot \sigma_0^2 / \sigma^2) \\ &= \text{Inv-}\chi^2(\sigma^2; 2(mN - qd + \nu_{\sigma^2}), \hat{\sigma}^2(B, \Sigma) \cdot (mN + 1 - qd + \nu_{\sigma^2}) / (mN - qd + \nu_{\sigma^2})) \end{aligned} \quad (3.4b)$$

$$\hat{\sigma}^2(B, \Sigma) = \frac{\text{tr}\left\{ R(B, \Sigma)^{-1} \left[ Y - A\hat{X}(B, \Sigma)\Phi \right] \left[ Y - A\hat{X}(B, \Sigma)\Phi \right]^H \right\} + \nu_{\sigma^2} \cdot \sigma_0^2}{mN + 1 - qd + \nu_{\sigma^2}}. \quad (3.4c)$$

$$H^{(i)} = P^{(i)} \left( B^{(i)} \right)^H \left( Y - AX^{(i)}\Phi \right) \quad (3.7c)$$

$$(\sigma^2)^{(i)} = \left\{ \text{tr} \left[ \left( \left( Y - AX^{(i)}\Phi - B^{(i)} H^{(i)} \right) \cdot \left( Y - AX^{(i)}\Phi \right)^H \right) + \nu_{\sigma^2} \cdot \sigma_0^2 \right] / \right. \\ \left. (mN + 1 - qd + \nu_{\sigma^2}) \right\} \quad (3.7d)$$

$$T^{(i)} = H^{(i)} (H^{(i)})^H + N \cdot (\sigma^2)^{(i)} \cdot P^{(i)} \quad (3.7e)$$

and update the interference parameters as follows:

$$\Sigma^{(i+1)} = \frac{1}{N + r + \nu_{\Sigma}} \cdot \left\{ \frac{T^{(i)}}{(\sigma^2)^{(i)}} + (\nu_{\Sigma} - r) \cdot A_0 \right\} \quad (3.8a)$$

$$B^{(i+1)} = \left[ \left( Y - AX^{(i)}\Phi \right) \left( H^{(i)} \right)^H + (\sigma^2)^{(i)} M_B \Gamma_B^{-1} \right] \\ \cdot \left[ T^{(i)} + (\sigma^2)^{(i)} \Gamma_B^{-1} \right]^{-1}. \quad (3.8b)$$

The above iteration is performed until the unknown parameters  $\xi^{(i)}$  do not change significantly, between two consecutive cycles. Each cycle requires one  $q \times q$  and two  $r \times r$  matrix inversions. Hence, this method is computationally efficient in applications with the signal and interference ranks  $q$  and  $r$  much smaller than  $m$ , such as STAP [6]–[8], [18] where  $m$  is the product of the numbers of sensors and pulse returns, see also [4] and [5]. Our ICM algorithm also provides estimates of the interference signals  $H$ , see (3.7c). Observe the intuitively appealing weighted-average forms of the expressions (3.7d), (3.8a), and (3.8b), where the weights of the prior terms are dictated by the prior degrees of freedom  $\nu_{\sigma^2}$ ,  $\nu_{\Sigma}$  and precision  $\Gamma_B^{-1}$ , respectively.

In the following, we describe initialization of the ICM iteration and AMF signal detection.

#### A. Initialization and Interference Rank Estimation

Our initialization scheme is based on the matrix  $Y[I_N - \Phi^H(\Phi\Phi^H)^{-1}\Phi]Y^H$ , which is complex Wishart with  $N - d$  degrees of freedom and scale  $\mathcal{R}(B, \Sigma, \sigma^2)$ . (This is a standard result in multivariate analysis of variance (MANOVA) whose real-Wishart version is given in, e.g., [28, Ch. 2.2].) Then

$$S = \frac{1}{N - d} \cdot Y \left[ I_N - \Phi^H (\Phi\Phi^H)^{-1} \Phi \right] Y^H \quad (3.9a)$$

is an unbiased estimator of  $\mathcal{R}(B, \Sigma, \sigma^2)$  with eigenvalue decomposition

$$S = [\mathbf{u}(1), \mathbf{u}(2) \cdots \mathbf{u}(m)] \text{diag} \{ \lambda_1, \lambda_2, \dots, \lambda_m \} \\ \cdot [\mathbf{u}(1), \mathbf{u}(2) \cdots \mathbf{u}(m)]^H \quad (3.9b)$$

where  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_m \geq 0$  are the eigenvalues and  $\mathbf{u}(1), \mathbf{u}(2) \cdots \mathbf{u}(m)$  are the corresponding eigenvectors of  $S$ . Then, define

$$\tilde{\sigma}^2 = \frac{\sum_{i=r+1}^{\min\{m, N-d\}} \lambda_i}{\min\{m, N-d\} - r} \quad (3.9c)$$

and initialize the iteration (3.7), (3.8) using

$$B^{(0)} = [\mathbf{u}(1) \cdots \mathbf{u}(r)], \quad \Sigma^{(0)} = \frac{\text{diag} \{ \lambda_1, \lambda_2, \dots, \lambda_r \}}{\tilde{\sigma}^2} - I_r. \quad (3.9d)$$

The above estimates can be computed efficiently using the algorithms in [11] and should be utilized when diffuse priors are employed for the unknown parameters; otherwise, prior information should be incorporated into the initial values as well.

**Interference Rank Estimation:** If the interference rank  $r$  is *not known*, we estimate it in the initialization stage using the minimum-description-length (MDL) criterion (derived along the lines of [29]) as shown in (3.10) at the bottom of the page, where  $n_{\text{MIN}} = \min\{m, N - d\}$  and  $n_{\text{MAX}} = \max\{m, N - d\}$ . If  $n_{\text{MIN}}$  is extremely small, the MDL approach performs poorly and better estimation of  $r$  can be achieved using statistical eigeninference in [30] and [31].

Here, we adopt a “plug-in” approach: once we estimate  $r = r_{\text{MDL}}$ , we treat it as if were known and proceed with the initialization (3.9c), (3.9d) and ICM iteration.

#### B. Adaptive- Matched-Filter Signal Detection

We propose the following AMF detector for testing  $H_0 : X = 0_{q \times d}$  (signal absent) versus the alternative  $H_1 : X \neq 0_{q \times d}$  (signal present):

Compare the test statistic  $\text{AMF}(\xi^{(\infty)})$  with a threshold  $\tau$  and declare the presence of signal if  $\text{AMF}(\xi^{(\infty)}) > \tau$ , where  $\text{AMF}(\xi) = \text{tr} \{ X^H A^H \mathcal{R}(B, \Sigma, \sigma^2)^{-1} A X \Phi \Phi^H \}$  is the matched-filter test statistic for the above testing problem assuming that the parameters  $\xi$  are *known*. (The AMF approach to signal detection was introduced in [32] and [33].) Here,  $\xi^{(\infty)}$  denotes the estimate of  $\xi$  obtained upon convergence of the ICM algorithm; consequently, we refer to the proposed detector as the *AMF ICM detector*. Note that  $\text{AMF}(\xi^{(\infty)})$  can be further simplified using (3.6) [see also (3.7a)]:

$$\text{AMF} \left( \xi^{(\infty)} \right) = \frac{1}{(\sigma^2)^{(\infty)}} \cdot \text{tr} \left\{ \left( X^{(\infty)} \right)^H \cdot \left[ A^H A - A^H B^{(\infty)} P^{(\infty)} \left( B^{(\infty)} \right)^H A \right] X^{(\infty)} \Phi \Phi^H \right\}. \quad (3.11)$$

## IV. NUMERICAL EXAMPLES

Consider an  $m = 50$ -sensor uniform linear antenna array with array response

$$\mathbf{a}(\varphi) = [1, \exp(-j\varphi), \exp(-2j\varphi), \dots, \exp(-(m-1)j\varphi)]^T \quad (4.1)$$

where  $\varphi = 2\pi\Delta \sin \theta / \lambda$ . Here, “ $T$ ” denotes a transpose,  $\Delta$  the interelement spacing,  $\theta$  the angle of arrival, and  $\lambda$  the wavelength of the center frequency. We focus on estimating and detecting the complex amplitude  $X = x$  of a planewave signal arriving from direction  $\varphi_s$ , implying that  $q = d = 1$  and signal array and temporal responses are vectors:  $A = \mathbf{a}(\varphi_s)$  and  $\Phi = [\phi(1), \phi(2) \cdots \phi(N)] = [1, 1, \dots, 1]$ . We have simulated  $r = 6$  independent Gaussian interference sources at angles  $\varphi \in \{-0.8\pi, -0.4\pi, 0.2\pi, 0.5\pi, 0.7\pi, 0.9\pi\}$  with 0-, 40-, 30-, 20-, 40-, and 10-dB interference-to-white-noise ratios (respectively), similar to the simulated interference model in [34].

$$r_{\text{MDL}} = \arg \max_{r \in \{1, 2, \dots, n_{\text{MIN}} - 1\}} \left\{ n_{\text{MAX}} \cdot \left( \sum_{i=r+1}^{n_{\text{MIN}}} \ln \lambda_i \right) - (n_{\text{MIN}} - r) n_{\text{MAX}} \ln \left( \frac{\sum_{l=r+1}^{n_{\text{MIN}}} \lambda_l}{n_{\text{MIN}} - r} \right) - \frac{1}{2} r (2n_{\text{MIN}} - r) \ln n_{\text{MAX}} \right\} \quad (3.10)$$

We have chosen a 0-dB signal level and direction  $\varphi_s = -0.81\pi$  satisfying  $\mathbf{a}(\varphi_s)^H [I_m - B(B^H B)^{-1} B^H] \mathbf{a}(\varphi_s) / \mathbf{a}(\varphi_s)^H \mathbf{a}(\varphi_s) = 0.19$ , implying that, in this case, interference “nulling” leads to a significant signal-power reduction [34]. Here, the interference-to-white-noise ratios are the diagonal elements of  $\Sigma$ , and the signal level is  $\text{abs}(x/\sigma)$ , where  $\text{abs}(\cdot)$  denotes absolute value.

**Diffuse Prior Specifications:** We have selected diffuse prior pdf’s for the unknown parameters with the following:

- $\Gamma_B^{-1} = 0_{r \times r}$  (corresponding to the “flat” Jeffreys’ noninformative prior for  $B$ );
- $\nu_\Sigma = r + 1 = 7$  and  $\Lambda_0 = \Sigma^{(0)}$  [see (3.9d) and footnote 4];
- $\nu_{\sigma^2} = 2$  and  $\sigma_0^2 = (\nu_{\sigma^2} - 1) / \nu_{\sigma^2} \cdot \tilde{\sigma}^2 = \tilde{\sigma}^2 / 2$  [yielding the mean of  $\pi(\sigma^2)$  equal to  $\tilde{\sigma}^2$ , see (3.9c) and footnote 3].

Under this prior model, the ICM estimates  $\xi^{(\infty)}$  of the unknown parameters are approximately equal to their (non-Bayesian) ML estimates. Consequently, the mean-square error (MSE) performance of  $X^{(\infty)}$  is bounded from below by the CRB derived in [15, App. 4.D] and [17, App. A]. Using [27, eq. (1.25) at p. 338], we simplify the trace of this CRB to

$$\text{CRB}(x) = \sigma^2 / \left\{ \left[ \mathbf{a}(\varphi_s)^H \mathbf{a}(\varphi_s) - \mathbf{a}(\varphi_s)^H B (\Sigma B^H B + I_r)^{-1} \Sigma B^H \mathbf{a}(\varphi_s) \right] \cdot \sum_{t=1}^N |\phi(t)|^2 \right\}. \quad (4.2)$$

**Interference Rank Estimation:** In all the examples, we employ the “plug-in” estimate  $r_{\text{MDL}}$  in (3.10), computed in the initialization stage.

**Complex-Amplitude Estimation:** In the first simulation example, we study the performance of the proposed estimator of  $X = x$ . Our performance metric is the MSE of an estimator, calculated using 30 000 independent trials. (In each trial, we generated independent interference and noise realizations.) We compare the ICM estimator of  $x$  with the GMANOVA method for unstructured covariance matrix of interference and noise (see [14]–[16]):

$$\hat{x}_{\text{UC}} = \frac{\mathbf{a}(\varphi_s)^H S^{-1} \sum_{t=1}^N \mathbf{y}(t) \phi(t)^*}{\mathbf{a}(\varphi_s)^H S^{-1} \mathbf{a}(\varphi_s) \cdot \sum_{t=1}^N |\phi(t)|^2} \quad (4.3a)$$

where “ $*$ ” denotes complex conjugation and  $S$  has been defined in (3.9a). We need  $N \geq \text{rank}(\Phi) + m = 51$  for  $S$  to be invertible with probability one (see [14, eq. (4)]). When the number of snapshots  $N$  is smaller than this bound, we apply diagonal loading as follows (similar to [35], see also [34] and references therein):

$$\hat{x}_{\text{UC,DL}} = \frac{\mathbf{a}(\varphi_s)^H (S + \delta^2 I_m)^{-1} \sum_{t=1}^N \mathbf{y}(t) \phi(t)^*}{\mathbf{a}(\varphi_s)^H (S + \delta^2 I_m)^{-1} \mathbf{a}(\varphi_s) \cdot \sum_{t=1}^N |\phi(t)|^2} \quad (4.3b)$$

where the loading factors have been selected as  $\delta^2 \in \{\tilde{\sigma}^2, 3\tilde{\sigma}^2\}$ , corresponding to 0 and 5 dB above the estimated noise level (3.9c) [8]. In Fig. 1, we show normalized MSEs and corresponding CRBs for the following estimates of the complex signal amplitude: i)  $x^{(\infty)}$  obtained upon convergence of the ICM algorithm and ii)  $\hat{x}_{\text{UC}}$  in (4.3a) (requiring  $N \geq 51$ ) and  $\hat{x}_{\text{UC,DL}}$  in (4.3b), as functions of the number of snapshots  $N$ . The ICM algorithm converged within ten iteration steps. For small  $N$ ,  $x^{(\infty)}$  outperforms  $\hat{x}_{\text{UC,DL}}$  approximately by a factor of two (in terms of MSE). Note that the (unstructured) GMANOVA estimator  $\hat{x}_{\text{UC}}$  performs poorly.

**Signal Detection:** In the second set of simulations, we compare the adaptive detector in Section III-B with several existing methods. Our performance metric is the *average probability of detection*, where averaging is performed over the random interference and noise realizations as well as Gaussian complex-amplitude realizations following the  $\mathcal{N}_{1 \times 1}(x; 0, \sigma_x^2, 1)$  distribution. Define the output signal-to-noise ratio

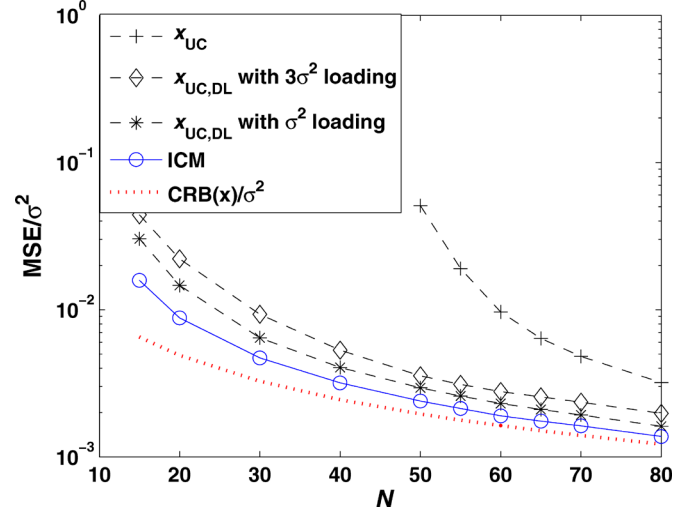


Fig. 1. Normalized MSEs and CRBs for the ICM, unstructured GMANOVA, and loaded unstructured GMANOVA estimators of the complex amplitude  $x$  as functions of the number of snapshots  $N$ , for an array with  $m = 50$  sensors.

(SNR) as  $\text{SNR}_o = \sigma_x^2 / \text{CRB}(x)$ . We compare the AMF ICM detector with the following:

- the *clairvoyant* detector, which assumes perfect knowledge of the interference and noise properties and compares  $\text{tr}[\hat{X}(B, \Sigma)^H A^H \mathcal{R}(B, \Sigma, \sigma^2)^{-1} A \hat{X}(B, \Sigma) \Phi \Phi^H]$  with a threshold (see also (3.3b) and (2.5b));
- generalized-likelihood ratio (GLR) and loaded GLR signal detectors for unstructured covariance matrix of interference and noise, which compare

$$\frac{\mathbf{a}(\varphi_s)^H S^{-1} \mathbf{a}(\varphi_s)}{\mathbf{a}(\varphi_s)^H [(1/N) \cdot YY^H]^{-1} \mathbf{a}(\varphi_s)}$$

and

$$\frac{\mathbf{a}(\varphi_s)^H (S + \delta^2 I_m)^{-1} \mathbf{a}(\varphi_s)}{\mathbf{a}(\varphi_s)^H [(1/N) \cdot YY^H + \delta^2 I_m]^{-1} \mathbf{a}(\varphi_s)}$$

(respectively) with appropriate thresholds, along the lines of [14, eq. (8)] and [34] and [35].

Fig. 2 shows the performances of the above detectors as functions of  $\text{SNR}_o$  for fixed false-alarm probability  $P_{\text{FA}} = 10^{-2}$ . The average detection performance of the clairvoyant detector can be computed analytically and is given by  $P_{d,\text{av}} = P_{\text{FA}}^{(1+\text{SNR}_o)^{-1}}$ . The AMF ICM detector outperforms other adaptive methods, with significant performance improvement for  $N = 20$ . As expected, the unstructured GLR detector performs poorly. Note that  $N = 20$  corresponds to the increasingly important “snapshot-constrained” scenario [4], [5].

Clearly, the proposed estimators and detectors will achieve better performance (compared with that shown in this section) if we utilize informative priors for the unknown interference and noise parameters.

## V. CONCLUDING REMARKS

We proposed an ICM algorithm for Bayesian estimation of complex signal amplitudes in low-rank interference and an adaptive signal detector based on the ICM estimates of the signal amplitudes and interference and noise parameters.

Interestingly, the initialization method in Section III-A is the main source of computational complexity since it requires eigenvalue

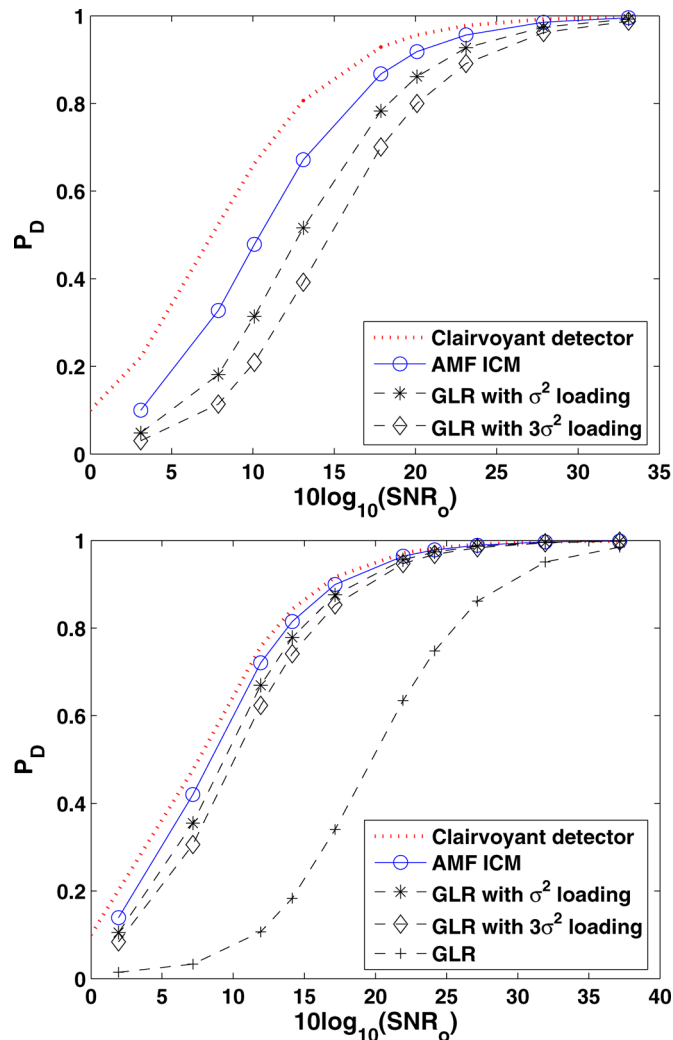


Fig. 2. Detection probabilities of the clairvoyant, AMF, GLR and loaded GLR detectors as functions of the output SNR in decibels for (top)  $N = 20$  and (bottom)  $N = 55$  snapshots, assuming  $P_{FA} = 10^{-2}$  and an array with  $m = 50$  sensors.

decomposition of a (potentially large)  $m \times m$  matrix. [The diagonal-loading approaches also require this decomposition to obtain the loading factor; see (3.9c).] If the array size  $m$  is large, then the proposed ICM algorithm and AML detector will be computationally efficient compared with the existing methods. Our approach also provides a framework for avoiding the complex initialization step, in particular, utilizing sequential-Bayesian concepts will allow *interference tracking*.

Further research will include: developing alternative initialization approaches and sequential-Bayesian methods for interference tracking (possibly using the “blending” ideas in [18]) and extensions to the STAP scenarios and comparison with related methods, such as the parametric adaptive matched filter (PAMF) in [36] and [37]. It is also of interest to compute analytical false-alarm and detection probability expressions for the proposed adaptive detector and to study Bayesian performance measures [38].

ACKNOWLEDGMENT

The authors are grateful to the anonymous reviewers for helpful comments and to N. Raj Rao from Massachusetts Institute of Technology, Cambridge, for insightful discussion and for bringing the rank-estimation approach in [30] and [31] to our attention.

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## Optimum Filtering for Energy-Spreading Transform-Based Equalization

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**Abstract**—In this correspondence, an improved scheme for energy spreading transform (EST) based equalization is proposed. In the improved scheme, optimal frequency- and time-domain filters that maximize signal-to-interference-noise ratio (SINR) are employed to enhance the performance. The performance improvement of optimal filtering is as large as 7 dB for some channels compared with the original scheme.

**Index Terms**—Energy spreading transform, equalization, iterative methods, signal processing for communications.

### I. INTRODUCTION

The iterative equalization based on energy spreading transform (EST) introduced in [1] has shown to be a promising technique that can effectively resolve severe intersymbol interference (ISI) with low computational complexity. It reaches the matched filter bound (MFB) for a system with signal-to-noise ratio (SNR) above a threshold.

In the original scheme, conventional minimum mean-square error (MMSE) equalization [2] is used at the first iteration, and matched filter (in frequency-domain) and the corresponding interference canceller (in time-domain) are employed for the rest of the iterations. The equalization proposed in this correspondence employs optimal frequency- and time-domain filters that maximize signal-to-interference-noise ratio (SINR) at each iteration to enhance the performance. From this point of view, the equalization filters used in [1] are an approximation of the optimal filters.

The rest of the correspondence is organized as follows. In Section II, we describe the improved equalization and analyze its asymptotic performance. We provide simulation results in Section IV. Finally, we draw conclusions in Section V.

### II. IMPROVED EQUALIZATION BASED ON EST

Fig. 1 shows the block diagram of the iterative equalization based on EST, which was first introduced in [1]. A block of transmit symbols  $\mathbf{x} = [x_0, x_1, \dots, x_{N-1}]^T$  with an average power  $\sigma_x^2$  is transformed to  $\tilde{\mathbf{x}} = [\tilde{x}_0, \tilde{x}_1, \dots, \tilde{x}_{N-1}]^T$  by an EST, where the superscript  $T$  denotes transpose and  $N$  is the block size. An EST is an orthonormal transform that uniformly spreads a symbol energy over the entire symbol block. Denote  $\mathbf{E} \in \mathbb{C}^{N \times N}$  to be an EST matrix. The ideal EST is an EST that has perfect spreading in both the time and frequency domain:

$$|(\mathbf{E})_{l,n}| = |(\mathbf{F}\mathbf{E})_{l,n}| = \frac{1}{\sqrt{N}} \quad (1)$$

and whose phase  $\angle(\mathbf{E})_{l,n}$  is pseudorandomly and symmetrically distributed in  $[-\pi, \pi]$  for  $0 \leq l, n \leq N-1$ , where  $\mathbf{F} \in \mathbb{C}^{N \times N}$  is the normalized Fourier-transform matrix, that is,  $(\mathbf{F})_{l,n} = (1/\sqrt{N})e^{-j2\pi ln/N}$ .

Manuscript received October 14, 2005; revised May 15, 2006. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Xiaodong Wang.

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Digital Object Identifier 10.1109/TSP.2006.887143