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Dynamic Shadow-Power Estimation for Wireless Communications

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***Abstract*—We present a sequential Bayesian method for dynamic estimation and prediction of local mean (shadow) powers from instantaneous signal powers in composite fading-shadowing wireless communication channels. We adopt a Nakagami-m fading model for the instantaneous signal powers and a first-order autoregressive [AR(1)] model for the shadow process in decibels. The proposed dynamic method approximates predictive shadow-power densities using a Gaussian distribution. We also derive Cramér–Rao bounds (CRBs) for stationary lognormal shadow powers and develop methods for estimating the AR model parameters. Numerical simulations demonstrate the performance of the proposed methods.**

***Index Terms*—Composite gamma-lognormal fading channels, dynamic shadow-power estimation, lognormal shadowing, Nakagami-m sequential Bayesian estimation.**

I. Introduction

In wireless communications, the ability to accurately estimate and predict local-mean (shadow) powers is instrumental for handoff,[1] channel access, power control, and adaptive modulation: The more accurately we estimate the local-mean signal level, the more efficiently we can perform these functions [1]–[8]. For example, the analysis of power-control algorithms for CDMA systems in [5] shows that reducing the shadow-power estimation error by 1 dB leads to a significant increase in achievable forward-link capacity (see also [2]). Several approaches to shadow-power estimation have been proposed [1]–[3], [7]–[9]. Window-based estimators in, e.g., [1, ch. 12.3], [3], and [7]–[9], are designed assuming constant shadow power over the duration of an averaging window. A Kalman-filter-based power estimation and prediction algorithm is developed in [2] for the composite Rayleigh-lognormal scenario and shown to meet or exceed the performance of window-based approaches. However, this method does not account for the non-Gaussian nature of the received log-powers in wireless radio environments. Recently, *sequential Bayesian methods* have attracted considerable attention due to their ability to overcome the limitations of the Kalman filter and successfully cope with non-Gaussian and nonlinear estimation problems.[2] In this correspondence (see also [16]), we develop a sequential Bayesian algorithm for estimating and predicting the shadow powers in composite fading-shadowing channels with a Nakagami-m component[3] and a shadowing component that follows a first-order autoregressive [AR(1)] random process. For stationary local-mean powers, we develop a nondynamic forward–backward (FB) algorithm for their estimation, as well as methods for estimating the model (AR and Nakagami-m) parameters.

We introduce the measurement model, derive sequential Bayesian and FB estimators (see Sections II-A and B), and compute Cramér–Rao bounds (CRBs) for the shadow powers (see Section II-C). In Section III, we propose methods for model parameter estimation. In Section IV, the accuracy of the proposed methods is evaluated using numerical simulations. Concluding remarks are given in Section V.

II. Measurement Model and Shadow Power Estimation

We describe a model for received-power fluctuations as a mobile subscriber moves through a wireless cellular radio environment. Passing the received signal through square-law envelope detector and amplifier (see, e.g., [7, Fig. 1] and [6]) and sampling the amplifier output yields a discrete-time sequence $y_k,\ k = 1, 2, \ldots$ of *instantaneous signal powers*.[4] We model y_k as the product of *mutually independent* fading and shadowing components [1, ch. 2.4.2], [2], [7], [8]

$$y_k = \chi_k \cdot 10^{\frac{\beta_k}{10}} \tag{2.1a}$$

where χ_k is the power fluctuation due to multipath fading, and β_k is the local-mean (shadow) power fluctuation in decibels. We assume that χ_k are independent and identically distributed (i.i.d.) gamma random variables with mean one, having the probability density function (pdf)

$$p_\chi(\chi_k; m) = \frac{m^m \chi_k^{m-1}}{\Gamma(m)} \cdot \exp(-m\chi_k) \tag{2.1b}$$

where $\Gamma(\cdot)$ denotes the gamma function, and m the denotes Nakagami-m fading parameter. (The fading samples χ_k are approximately independent if the sampling interval is large enough; see also the discussion in Section IV.) Finally, we model β_k as a first-order AR(1) random process

$$\beta_k = \alpha_k \beta_{k-1} + \omega_k \tag{2.1c}$$

where ω_k are independent zero-mean random variables with variances $\sigma^2_{\omega,k}$. The AR(1) model (2.1c) is widely used to describe the correlation of the shadow process β_k (see, e.g., [2], [6]–[8], and [17]). Note that AR shadow modeling is different from AR channel modeling (see the discussion in [2, Sect. IV]). Here, we estimate and predict the *unknown* shadow powers β_k, assuming that the model parameters (Nakagami-m parameter, AR coefficients α_k, and variances $\sigma^2_{\omega,k}$) are *known*. An extension to the scenario where the model parameters are unknown is considered in Section III.

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[1]For example, effective implementations of soft handoff for code-division multiple access (CDMA) cellular systems are based on shadow-power estimates, leading to extended cell coverage and increased reverse-link capacity [4].

[2]In wireless communications, recursive Bayesian methods have been applied to channel tracking [11], blind detection, equalization, and deconvolution [12], [13], mobility tracking [14], and impulsive interference identification [15].

[3]The Nakagami-m fading model is fairly general: It includes Rayleigh fading as a special case and can be used to closely approximate Ricean and Nakagami-q (Hoyt) fading scenarios (see [10, ch. 2.2.1.4]).

[4]We neglect the effects of additive noise in the derivation of the proposed methods and assume that the instantaneous signal powers y_k are accurately measured (see also [2], [3], and [7]–[9]). However, the presence of noise is considered in our numerical simulations (see Fig. 7 in Section IV).

A. Sequential Bayesian shadow-power estimation

We now derive a sequential Bayesian method for shadow-power estimation and prediction. Note that we have not specified the distributional form of the random variables ω_k apart from their first two moments; hence, the distribution of the shadow process $\beta_k, k = 1, 2, \ldots$ is also not fully specified. (For a *fully specified* pdf of β_k, the recursion for computing its prediction and filtering densities is given in Appendix A.) Denote by μ_k and c_k the posterior mean and variance of β_k given the set $\boldsymbol{y}_{1:k} = \{y_1, y_2, \ldots, y_k\}$ of all instantaneous powers until time k. Immediately before observing y_k, all currently available information is described by the mean μ_{k-1} and variance c_{k-1}. At time $k = 1$, these are the starting values μ_0 and c_0 and, for all other k, will come from the *posterior (filtering) distribution* of β_{k-1} given $\boldsymbol{y}_{1:(k-1)}$, denoted by $[\beta_{k-1}|\boldsymbol{y}_{1:(k-1)}]$. Using the AR(1) model in (2.1c), we compute the mean b_k and variance r_k of the *prior (predictive) distribution* $[\beta_k|\boldsymbol{y}_{1:(k-1)}]$

$$b_k = \alpha_k \mu_{k-1}, \quad r_k = \alpha_k^2 c_{k-1} + \sigma_{\omega,k}^2. \tag{2.2}$$

Since $[\beta_k|\boldsymbol{y}_{1:(k-1)}]$ is specified only through the above moments, we are free to choose the form of this distribution as long as it is consistent with (2.2); here, we adopt the Gaussian pdf with mean and variance given in (2.2)

$$\left[\beta_k|\boldsymbol{y}_{1:(k-1)}\right] \sim g(\beta_k; b_k, r_k) = \frac{1}{\sqrt{2\pi r_k}} \cdot e^{-(\beta_k - b_k)^2/(2r_k)}. \tag{2.3}$$

In other words, we approximate the "exact" (and generally *analytically intractable*) predictive distribution in (A.1a) in Appendix A using the above Gaussian pdf, which leads to the *posterior updating equations* in (2.4a) and (2.4b), shown at the bottom of the page, where

$$v_l(b_k, r_k) = 10^{(\sqrt{2r_k}\cdot x_l + b_k)/10}$$

and an approximate expression for $\mathrm{E}_{\beta|\boldsymbol{y}}\left[\beta_k^2|\boldsymbol{y}_{1:k}\right]$ is in (2.4c), shown at the bottom of the page. The posterior updating equations are derived as the mean and variance of $[\beta_k|\boldsymbol{y}_{1:k}]$, where $[\beta_k|\boldsymbol{y}_{1:k}]$ is obtained by substituting the approximation (2.3) into the "exact" filtering-density expression (A.1b) in Appendix A. The approximate expressions (2.4a) and (2.4c) follow by using the Gauss–Hermite quadrature (GHQ) to numerically evaluate the above conditional expectations. Here, L is the quadrature order (determining approximation accuracy), and x_l, h_{x_l}, $l = 1, \ldots, L$ are the GHQ abscissas and weights, tabulated in, e.g., [19]. The GHQ approximation has been used in [20] for nonlinear state estimation in stochastic dynamical systems.

To summarize, we have developed a sequential Bayesian method for dynamic estimation and prediction of shadow powers whose predictive pdfs are approximated using a Gaussian distribution; the proposed recursion alternates between

- the prior cascade equations (2.2);
- posterior updating equations (2.4).

Assuming that instantaneous signal powers until time k are available, our *estimator* of β_k is given by (2.4a), and the *one-step predictor* of β_{k+1} is $b_{k+1} = \alpha_{k+1}\mu_k$ [see (2.2)].

B. Forward–Backward Estimation of Stationary Shadow Powers

Assume that the AR coefficients α_k and variances $\sigma_{\omega,k}^2$ are constant (independent of k) in the interval $\{1, 2, \ldots, K\}$, i.e.,

$$\alpha_k = \alpha \in (-1, 1), \quad \sigma_{\omega,k}^2 = \sigma_\omega^2 \tag{2.5}$$

for $k = 1, 2, \ldots, K$, implying *stationarity* of the shadow process β_k. Then, the variance of β_k is

$$\sigma_\beta^2 = \frac{\sigma_\omega^2}{(1-\alpha^2)}. \tag{2.6}$$

We now present a *nondynamic (batch)* FB estimator of the stationary shadow powers. In addition to the "forward" recursion described in Section II-A, we also apply the proposed recursion "backward" to the observations arranged in the reverse order: $y_K, y_{K-1}, \ldots, y_1$. Hence, an improved shadow-power estimator is obtained by running *both* recursions and *averaging* the obtained forward and backward estimates of $\beta_1, \beta_2, \ldots, \beta_K$.

C. CRB for Stationary Lognormal Shadow Powers

We derive the *Bayesian Cramér–Rao bound* for the shadow-power vector $\boldsymbol{\beta} = [\beta_1, \beta_2, \ldots, \beta_K]^T$ assuming Gaussian $\boldsymbol{\beta}$ (lognormal shadowing), known model parameters, and stationary shadow powers

$$\mathrm{CRB}_\beta = \mathcal{I}_\beta^{-1} \tag{2.7}$$

where $\mathcal{I}_\beta$ is the Bayesian Fisher information matrix. (For the definition and properties of the Bayesian Cramér–Rao bound, see [21, ch. 2.4].) Here, $\mathcal{I}_\beta$ is a tridiagonal matrix whose sub- and super-diagonal elements are equal to $-\alpha/\sigma_\omega^2$, and its diagonal elements are equal to $m(\ln 10/10)^2 + (1+\alpha^2)/\sigma_\omega^2$ for $k \in \{2, 3, \ldots, K-1\}$ and $m(\ln 10/10)^2 + 1/\sigma_\omega^2$ for $k \in \{1, K\}$. The derivation of $\mathcal{I}_\beta$ is outlined in Appendix B. An extension of the above CRB results to the nonstationary scenario is straightforward. Assuming stationarity and a large number of samples K approximating $\mathcal{I}_\beta$ with a circulant matrix, we derive an approximate formula for the average CRB

$$\frac{\mathrm{tr}(\mathrm{CRB}_\beta)}{K} \approx \left\{ \left[m\left(\frac{\ln 10}{10}\right)^2 + \sigma_\beta^{-2}\cdot\frac{1-\alpha}{1+\alpha}\right] \cdot \left[m\left(\frac{\ln 10}{10}\right)^2 + \sigma_\beta^{-2}\cdot\frac{1+\alpha}{1-\alpha}\right]\right\}^{-\frac{1}{2}}. \tag{2.8}$$

$$\mu_k = \mathrm{E}_{\beta|\boldsymbol{y}}[\beta_k|\boldsymbol{y}_{1:k}] \approx \frac{\sum_{l=1}^{L} h_{x_l}\cdot(\sqrt{2r_k}\cdot x_l + b_k)\cdot\exp\left[-\frac{m y_k}{v_l(b_k,r_k)}\right]\cdot v_l(b_k,r_k)^{-m}}{\sum_{l=1}^{L} h_{x_l}\exp\left[-\frac{m y_k}{v_l(b_k,r_k)}\right]\cdot v_l(b_k,r_k)^{-m}} \tag{2.4a}$$

$$c_k = \mathrm{var}_{\beta|\boldsymbol{y}}[\beta_k|\boldsymbol{y}_{1:k}] = \mathrm{E}_{\beta|\boldsymbol{y}}\left[\beta_k^2|\boldsymbol{y}_{1:k}\right] - \mu_k^2, \tag{2.4b}$$

$$\mathrm{E}_{\beta|\boldsymbol{y}}\left[\beta_k^2|\boldsymbol{y}_{1:k}\right] \approx \frac{\sum_{l=1}^{L} h_{x_l}\cdot(\sqrt{2r_k}\cdot x_l + b_k)^2\cdot\exp\left[-\frac{m y_k}{v_l(b_k,r_k)}\right]\cdot v_l(b_k,r_k)^{-m}}{\sum_{l=1}^{L} h_{x_l}\exp\left[-\frac{m y_k}{v_l(b_k,r_k)}\right]\cdot v_l(b_k,r_k)^{-m}}. \tag{2.4c}$$

Small σ_β^2, large α (close to one), or large m lead to small average CRB and good estimation performance.

In the following, we consider the case where the model parameters α, σ_ω^2, and m are *unknown* and develop methods for their estimation when the shadow powers are stationary.

III. Estimating Unknown Model Parameters

We present an iterative *alternating-projection* method for *jointly* estimating the AR model parameters *and* shadow powers under the stationarity assumptions in (2.5): Iterate between the following two steps.

Step 1 (AML): Fix $\beta_1, \beta_2, \ldots, \beta_K$, and estimate α and σ_ω^2 using their *asymptotic maximum likelihood (AML) estimates* (see, e.g., [22, Ex. 7.18])

$$\widehat{\alpha} = \frac{\sum_{l=2}^{K} \beta_l \beta_{l-1}}{\sum_{k=1}^{K} \beta_k^2} \tag{3.1a}$$

$$\widehat{\sigma}_\omega^2 = (1 - \widehat{\alpha}^2) \cdot \frac{\sum_{k=1}^{K} \beta_k^2}{K}. \tag{3.1b}$$

Step 2 (FB): Fix α and σ_ω^2, and estimate $\beta_1, \beta_2, \ldots, \beta_K$ using the FB method in Section II-B.

shadow-power estimation for unknown AR model parameters is important in urban environments if the sampling period with which the measurements are collected is relatively large (see [2, Sec. IV]). The above iteration can be initialized using the instantaneous powers in decibels: $\beta_k^{\text{init}}(t) = (10/\ln 10) \cdot \ln y_k$, $k = 1, 2, \ldots, K$. Note that Step 2 requires the knowledge of the Nakagami-m fading parameter, which can be estimated separately using the method in [23], discussed briefly below.

Nakagami-m Parameter Estimation: In [23], we derive ML methods for estimating m from the instantaneous powers $y_1, y_2, \ldots, y_K$ under the *piecewise-constant* model for the shadow powers. In particular, $\beta_1, \beta_2, \ldots, \beta_K$ are assumed to be constant within intervals (windows) of length N but allowed to vary randomly from one interval to another. In [23], we have chosen $K = LN$ and $\beta_{(l-1)N+1} = \beta_{(l-1)N+2} = \ldots = \beta_{(l-1)N+N} = z_l$, where z_l, $l = 1, 2, \ldots, L$ are modeled as i.i.d. Gaussian random variables with unknown mean and variance.

Denote the estimates of $\beta_1, \beta_2, \ldots, \beta_K$ the above AML/FB iteration by $\widehat{\beta}_1, \widehat{\beta}_2, \ldots, \widehat{\beta}_K$. In the following, we utilize $\widehat{\beta}_1, \widehat{\beta}_2, \ldots, \widehat{\beta}_K$ to compute improved estimates of α and σ_ω^2 *estimated likelihood* (EL) approach.

A. EL Estimation of the AR Model Parameters

We now treat the estimates $\widehat{\beta}_1, \widehat{\beta}_2, \ldots, \widehat{\beta}_K$ as observations and estimate α and σ_ω^2 by maximizing the *estimated log-likelihood function*:[5]

$$L_{\text{EL}}(\alpha, \sigma_\omega) = \frac{1}{2}\ln(1-\alpha^2) - \frac{K}{2} \cdot \ln\left(2\pi\sigma_\omega^2\right) - \frac{\widehat{\beta}_1^2 + \widehat{\beta}_K^2}{2\sigma_\omega^2} - \frac{\alpha^2}{2\sigma_\omega^2} \cdot \left(\sum_{l=2}^{K-1} \widehat{\beta}_l^2\right) + \frac{\alpha}{\sigma_\omega^2} \cdot \left(\sum_{l=2}^{K} \widehat{\beta}_l \widehat{\beta}_{l-1}\right) \tag{3.2}$$

with respect to α and σ_ω^2. This maximization yields the EL estimates of α and σ_ω^2 and can be performed using alternating projections, as described below. We first estimate α for fixed σ_ω^2. Differentiating (3.2) with respect to α and setting the result to zero yields

$$-\alpha\sigma_\omega^2 - \alpha(1-\alpha^2) \cdot \left(\sum_{l=2}^{K-1} \widehat{\beta}_l^2\right) + (1-\alpha^2) \cdot \left(\sum_{l=2}^{K} \widehat{\beta}_l \widehat{\beta}_{l-1}\right) = 0 \tag{3.3a}$$

which can be solved by polynomial rooting. Note that the left-hand side of (3.3a) is positive at $\alpha = -1$ and negative at $\alpha = 1$, implying that we can always find a real root α above polynomial within the parameter space [satisfying $\alpha \in (-1, 1)$] for which the second derivative of (3.2) is negative. Consequently, we estimate α as the conforming root of (3.3a), which maximizes (3.2). We now fix α and estimate σ_ω^2. Maximizing (3.2) with respect to σ_ω^2 yields

$$\sigma_\omega^2 = \frac{1}{K} \cdot \left[\widehat{\beta}_1^2 + \widehat{\beta}_K^2 + (1+\alpha^2) \cdot \left(\sum_{l=2}^{K-1} \beta_l^2\right) - 2\alpha \cdot \left(\sum_{l=2}^{K} \beta_l \beta_{l-1}\right)\right]. \tag{3.3b}$$

To find the EL estimates of α and σ_ω^2 that *jointly* maximize (3.2), iterate between the polynomial-rooting-based estimation of α in (3.3a) and the estimation of σ_ω^2 in (3.3b) until convergence. After computing the EL estimates of α and σ_ω^2, we can apply the FB method to obtain improved *estimated-likelihood/forward-backward* (EL/FB) shadow-power estimates.

IV. Numerical Examples

We assess the estimation accuracy of the proposed methods and compare them with the existing techniques. The instantaneous powers y_k, $k = 1, 2, \ldots$ were simulated using a composite *gamma-lognormal* fading-shadowing scenario described by (2.1) with Gaussian w_k, $k = 1, 2, \ldots$. We also assume that the stationarity conditions (2.5) are satisfied. Our performance metric is the mean-square error (MSE) of an estimator, calculated using 4000 independent trials. The quadrature order of the Gauss–Hermite approximations in (2.4a) and (2.4c) was $L = 20$, unless specified otherwise (see Fig. 3). (When $L = 20$, the errors introduced by these approximations are negligible compared with the estimation errors due to randomness introduced by the measurement model.)

In the first set of simulations, we generated the simulated data using the measurement model in Section II. We selected $\alpha_k = \alpha = 0.9704$ and $\sigma_{\omega,k}^2 = \sigma_\omega^2 = 0.9318$, which are typical values in an urban environment obtained by choosing the *shadow standard deviation* $\sigma_\beta = 4$ dB and *effective correlation distance, mobile speed, and sampling interval* equal to $\xi_c = 10$ m, $v = 20$ km/h, and $T = 54$ ms.[6] Consider first the scenario where the model parameters are *known*. We applied the sequential Bayesian method in Section II-A to estimate and predict the unknown shadow powers; this method was initialized using the mean and variance of β_k: $\mu_0 = 0$ $c_0 = \sigma_\beta^2 = 16$. In Figs. 1 and 2, we show the MSEs (averaged over the K samples) for the sequential Bayesian estimator (2.4a) and one-step predictor for $m = 1$ (Rayleigh fading) and $m = 3$, respectively, as functions of the number of samples K. Figs. 1 and 2 also show the average MSEs for the Kalman-filter-based shadow-power estimators and predictors recently proposed in [2]. The method in [2] is derived by applying the Kalman filter to

[5]See [24, ch. 10.7] for the definition and properties of the estimated likelihood and [24, ch. 11.1] for the pdf of an AR(1) Gaussian random process.

[6]To compute α, we apply the following formula: $\alpha = \exp(-vT/\xi_c)$ (see, e.g., [2]); to compute σ_ω^2, we use (2.6).

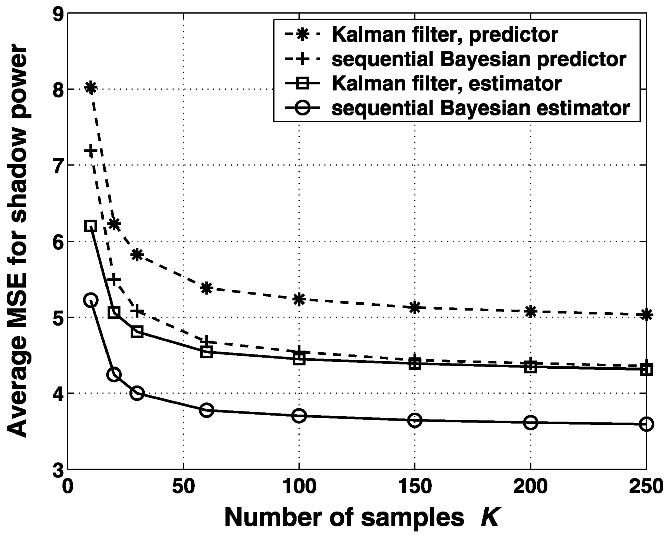

Fig. 1. Average MSEs for the sequential Bayesian and Kalman-filter-based estimators and predictors of the shadow powers as functions of K, assuming known model parameters and $m = 1$ (Rayleigh fading).

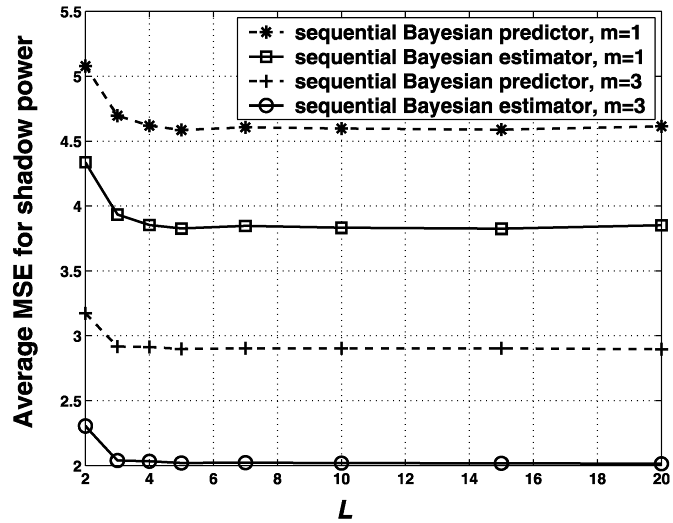

Fig. 3. Average MSEs for the sequential Bayesian estimator and predictor of the shadow powers as a function of the quadrature order L, for $K = 200$ and $m \in \{1, 3\}$.

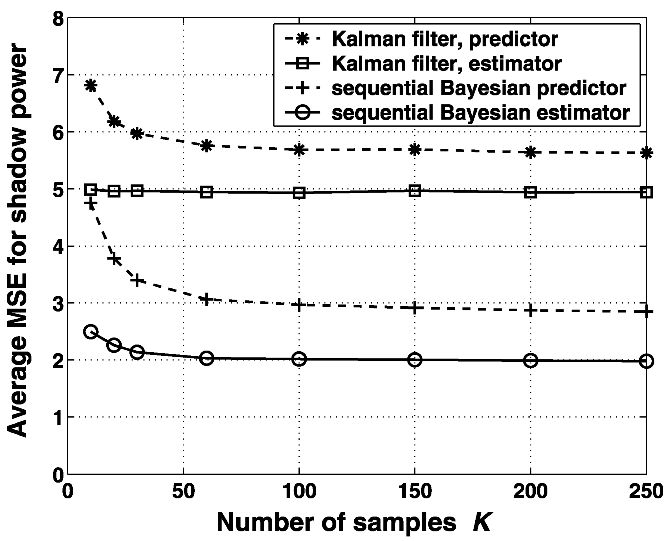

Fig. 2. Average MSEs for the sequential Bayesian and Kalman-filter-based estimators and predictors of the shadow powers as functions of K, assuming known model parameters and $m = 3$.

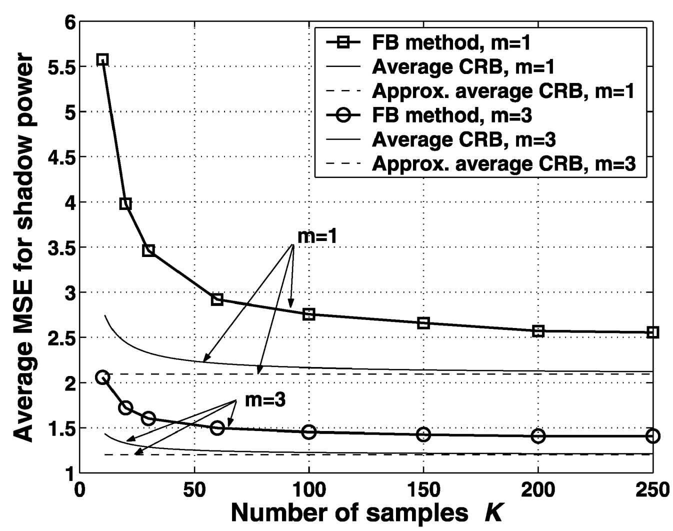

Fig. 4. Average MSEs and corresponding CRBs for the FB estimates of the shadow powers as functions of K, assuming known model parameters and $m \in \{1, 3\}$.

the log-domain model [obtained by taking the logarithm of (2.1a)], where the instantaneous signal power in decibels is decomposed into a sum of the shadowing component and the fading component. However, the fading component is non-Gaussian, and the Kalman filter ignores its distributional form, effectively approximating it with a Gaussian distribution. This is in contrast with the sequential Bayesian method in Section II-A, which utilizes the distribution of the fading component. The sequential Bayesian method outperforms the Kalman filter in both scenarios;[7] in the Rayleigh-fading case, the sequential Bayesian predictor performs as well as the Kalman-filter estimator (see Fig. 1). In terms of CPU time, the sequential Bayesian algorithm is approximately L times slower than the Kalman filter, where L denotes the quadrature order. In Fig. 3, we present the average MSEs for the sequential Bayesian estimator and predictor as functions of L, for $m \in \{1, 3\}$ and $K = 200$.

[7]Note that the Kalman filtering method in [2] was designed for the Rayleigh-fading scenario.

In this case, the error introduced by the integral approximations (2.4a) and (2.4c) affects the MSE curves only when very small quadrature orders ($L \leq 3$) are used. We also examine the performance of the non-dynamic FB method in Section II-B. Fig. 4 shows the average MSEs for the FB power estimates and corresponding average Bayesian CRBs as functions of K, where $m \in \{1, 3\}$. For large K, the average CRBs are well approximated by (2.8).

We now consider the scenario where the model parameters α, σ_ω^2, and m are *unknown*. Fig. 5 shows the average MSEs for the AML/FB and EL/FB shadow-power estimates as functions of K (see also Section III). The AML/FB method converged within 15 steps. In Fig. 6, we show the MSE for the estimator of m in [23] (using the window length $N = 5$) and the MSEs for the AML/FB and EL estimators of α and σ_ω^2 as functions of K. The EL method gives significantly better estimates of α compared with the AML/FB method. This, in turn, improves shadow-power estimation (see Fig. 5).

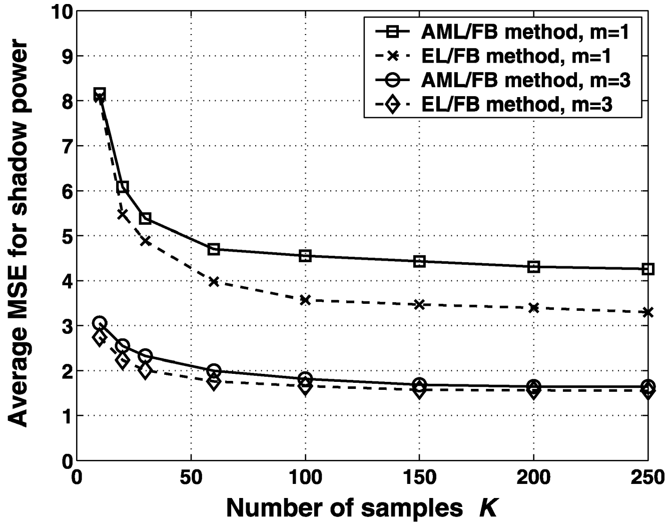

Fig. 5. Average MSEs for the AML/FB and EL/FB shadow-power estimators as functions of K for $m \in \{1, 3\}$.

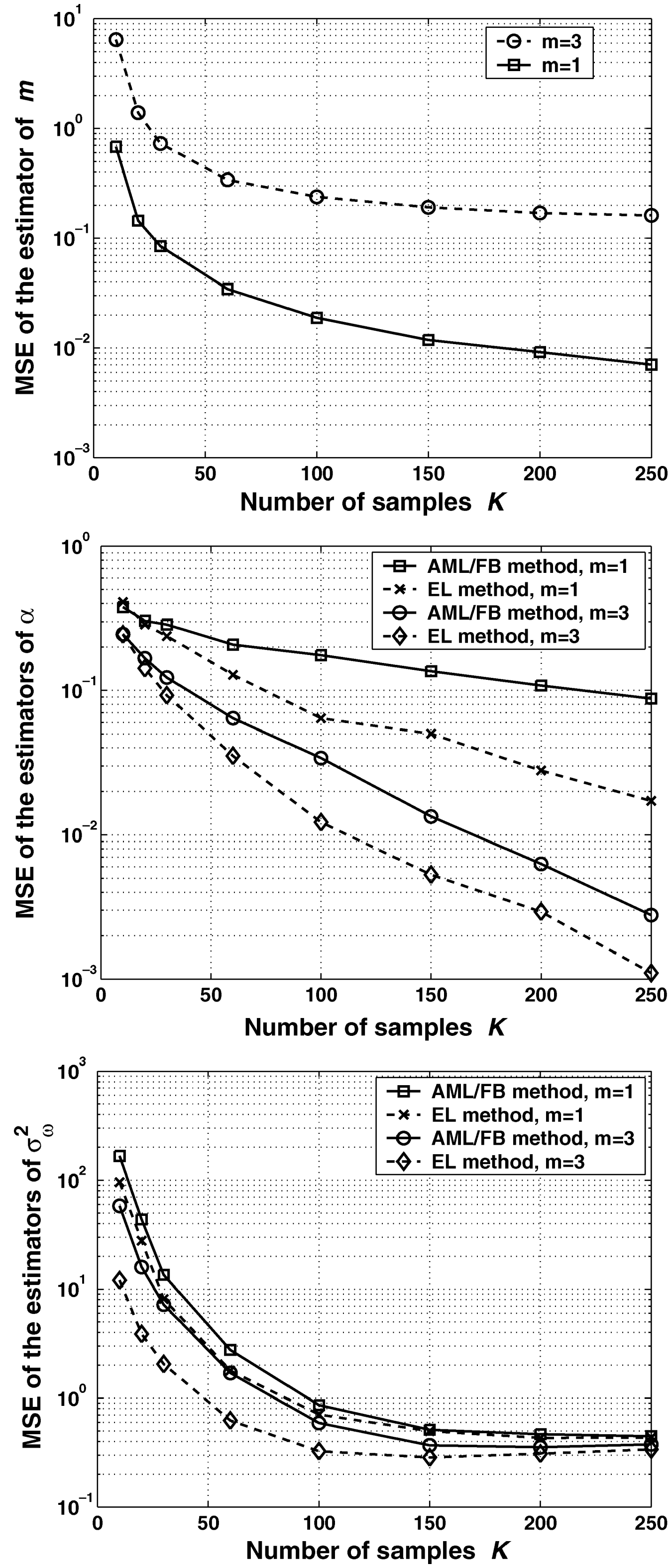

Fig. 6. MSEs for the AML/FB estimates of the model parameters (m, α, and σ_ω^2, respectively) as functions of K, for $m \in \{1, 3\}$.

Correlated Ricean Fading: In the second set of simulations, we consider a correlated noisy Ricean-fading scenario with known model parameters and received instantaneous signal powers y_k modeled as

$$y_k = \left| 10^{\frac{\beta_k}{20}} \cdot h_k + e_k \right|^2 \tag{4.1}$$

where the shadow process β_k is described in Section II, and two stationary circularly symmetric complex Gaussian random processes h_k and e_k model fading and noise effects, respectively. We assume that β_k, h_k, and e_k are mutually independent, e_k is a zero-mean white noise with variance σ^2, and the mean and autocovariance function of h_k are $\mathrm{E}[h_k] = \mu_h \cdot \exp[j(2\pi v_{\mathrm{LOS}} T/\lambda)k]$ and $\mathrm{E}[(h_k - \mathrm{E}[h_k])(h_l - \mathrm{E}[h_k])^*] = (1 - |\mu_h|^2) \cdot J_0((2\pi v T/\lambda) \cdot (k - l))$, respectively. Here, $0 \le |\mu_h| < 1$, corresponding to the Ricean $\mathbb{K}$ factor

$$\mathbb{K} = \frac{|\mu_h|^2}{1 - |\mu_h|^2} \tag{4.2}$$

and the autocovariance function of h_k follows the Jakes' model for uniformly distributed scatterers around the mobile, see e.g. [1]. Note that "*" denotes complex conjugation, $J_0(\cdot)$ the zeroth-order Bessel function of the first kind, v and v_{LOS} the magnitude and line-of-sight component of the mobile velocity, respectively, λ the wavelength corresponding to the carrier frequency, and T the sampling interval. We selected $v = 20$ km/h, $v_{\mathrm{LOS}} = 10$ km/h, $\xi_c = 10$ m, $\lambda = 1/3$ m, $\sigma_\beta = 4$ dB, and $\mathbb{K} = 4$. The Nakagami-m parameter was computed using the approximate formula in [10, eq. (2.26)]

$$m \approx \frac{(1 + \mathbb{K})^2}{1 + 2\mathbb{K}} = \frac{1}{1 - |\mu_h|^4} \tag{4.3}$$

which is approximately equal to 3 for the above choice of model parameters. In parts (a) and (b) of Fig. 7, we present the average MSEs for the sample-mean and uniformly minimum variance unbiased (UMVU) window-based estimators [1]–[3] as functions of the window length for (a) $\sigma^2 = 0$ (noiseless scenario) and (b) $\sigma^2 = 0.2$ (noisy scenario), assuming $T = 54$ ms (i.e., $\alpha = 0.9704$ and $\sigma_\omega^2 = 0.9318$; see footnote 6). Parts (c) and (d) of Fig. 7 show corresponding average MSEs obtained using a smaller sampling interval $T = 5$ ms. Fig. 7 also

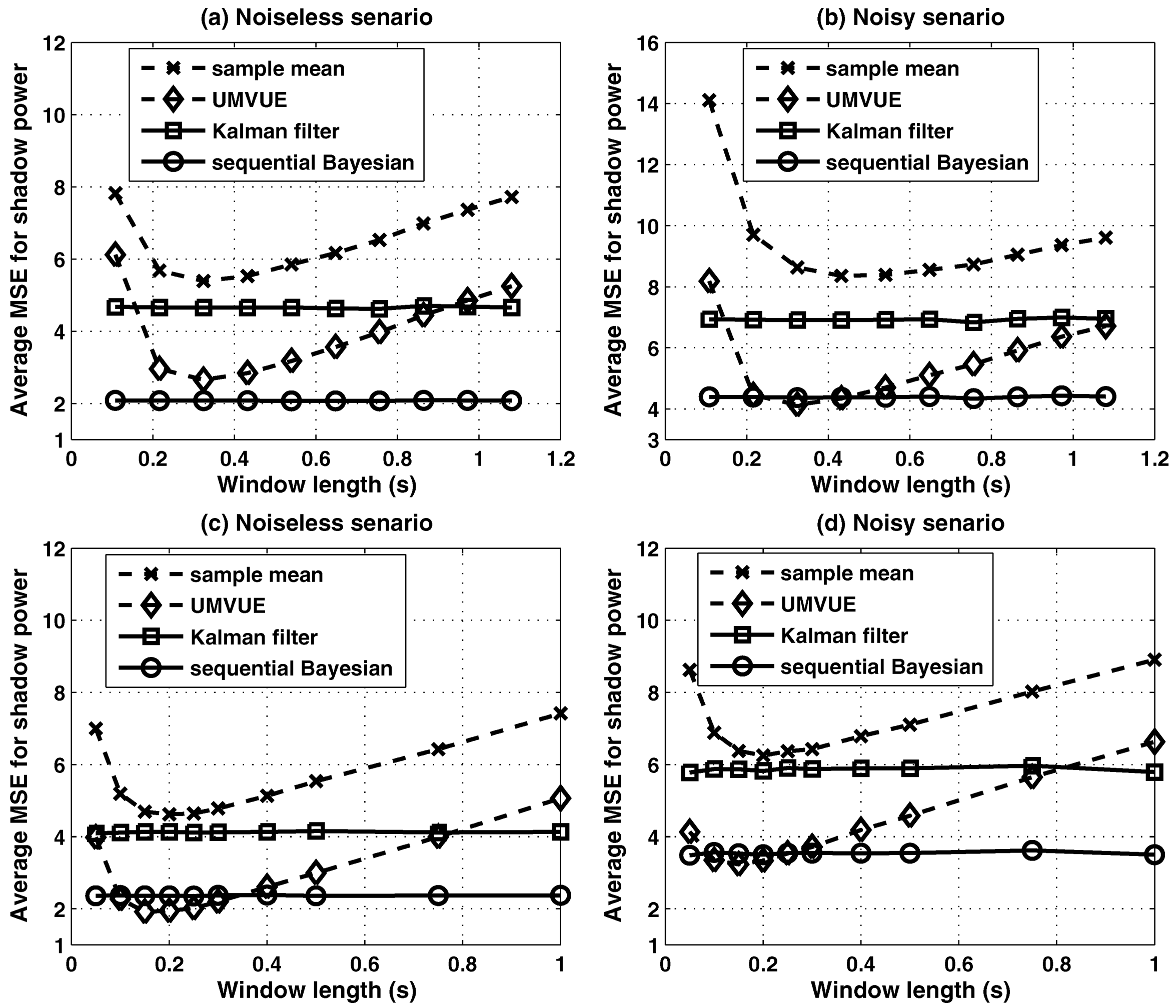

Fig. 7. Average MSEs for the sequential Bayesian, Kalman-filter, and window-based shadow-power estimators as functions of the window length, assuming correlated Ricean fading with (a) $\sigma^2 = 0$ and $T = 54$ ms, (b) $\sigma^2 = 0.2$ and $T = 54$ ms, (c) $\sigma^2 = 0$ and $T = 5$ ms, and (d) $\sigma^2 = 0.2$ and $T = 5$ ms.

shows the average MSE performances of the sequential Bayesian and Kalman-filter-based methods. If the fading component is not strongly correlated (large T), the sequential Bayesian estimator outperforms the Kalman-filter and window-based estimators. For strongly correlated fading (small T), the UMVU window-based method outperforms the sequential Bayesian and Kalman-filter-based methods if the window length is chosen correctly.

V. Concluding Remarks

We proposed a sequential Bayesian method for shadow-power estimation and prediction in composite fading-shadowing wireless communication channels with a Nakagami-m fading component and AR(1) shadowing component. For stationary shadow powers, we derived a nondynamic forward-backward power estimator, exact and approximate Bayesian CRBs, and methods for estimating the unknown model parameters. Further research will include developing shadow-power estimation methods that account for fading correlations and noisy instantaneous-power estimates.

Appendix A
Recursions for Computing the Prediction and Filtering Densities of β_k

We present general recursions for computing the prediction and filtering densities of β_k, assuming that both the *observation-model pdf* $p_{y|\beta}(y_k|\beta_k)$ and *Markov transition pdf* $p_{\beta_k|\beta_{k-1}}(\beta_k|\beta_{k-1})$ are available (see [18, eqs. (3.14) and (3.16)])

$$p_{\beta_k|\boldsymbol{y}_{1:(k-1)}}\left(\beta_k|\boldsymbol{y}_{1:(k-1)}\right) = \int p_{\beta_k|\beta_{k-1}}(\beta_k|\beta) p_{\beta_{k-1}|\boldsymbol{y}_{1:(k-1)}}\left(\beta|\boldsymbol{y}_{1:(k-1)}\right) d\beta \quad \text{(A.1a)}$$

$$p_{\beta_k|\boldsymbol{y}_{1:k}}(\beta_k|\boldsymbol{y}_{1:k}) = \frac{p_{y|\beta}(y_k|\beta_k) p_{\beta_k|\boldsymbol{y}_{1:(k-1)}}\left(\beta_k|\boldsymbol{y}_{1:(k-1)}\right)}{\int p_{y|\beta}(y_k|\beta) p_{\beta_k|\boldsymbol{y}_{1:(k-1)}}\left(\beta|\boldsymbol{y}_{1:(k-1)}\right) d\beta}. \quad \text{(A.1b)}$$

Under the measurement model in Section II, the observation-model pdf follows from (2.1a) and (2.1b). Furthermore, assuming lognormal shadowing (i.e., Gaussian β_k) and AR(1) model in (2.1c), the transition pdf is $p_{\beta_k|\beta_{k-1}}(\beta_k|\beta_{k-1}) = g(\beta_k; \alpha_k\beta_{k-1}, \sigma^2_{\omega,k})$. Under this scenario, (A.1a) and (A.1b) are analytically intractable.

APPENDIX B
FISHER INFORMATION MATRIX FOR STATIONARY SHADOW POWERS

We derive the Bayesian Fisher information matrix $\mathcal{I}_\beta$ in Section II-C. Under the stationarity assumptions in (2.5), the logarithm of the joint pdf of $\mathbf{y} = [y_1, y_2, \ldots, y_K]^T$ and $\boldsymbol{\beta}$ is

$$\begin{aligned} &L_c\left(m, \alpha, \sigma_\omega^2; \boldsymbol{y}, \beta\right) \\ &= Km \ln m + (m-1) \cdot \left(\sum_{k=1}^{K} \ln y_k\right) \\ &\quad - \frac{\ln 10}{10} \cdot m \cdot \left(\sum_{k=1}^{K} \beta_k\right) - m \sum_{k=1}^{K} y_k 10^{-\frac{\beta_k}{10}} \\ &\quad - K \ln \Gamma(m) - \frac{K}{2} \ln\left(2\pi\sigma_\omega^2\right) + \frac{1}{2} \ln(1-\alpha^2) \\ &\quad - \frac{\beta_1^2 + \beta_K^2}{2\sigma_\omega^2} - \frac{1+\alpha^2}{2\sigma_\omega^2} \cdot \left(\sum_{l=2}^{K-1} \beta_l^2\right) \\ &\quad + \frac{\alpha}{\sigma_\omega^2} \cdot \left(\sum_{l=2}^{K} \beta_l \beta_{l-1}\right). \end{aligned} \quad \text{(B.1)}$$

Differentiating (B.1) twice with respect to $\boldsymbol{\beta}$ and taking joint expectation with respect to $\boldsymbol{y}$ and $\boldsymbol{\beta}$ yields $\mathcal{I}_\beta$.

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Complex Approximation of FIR Digital Filters by Updating Desired Responses

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***Abstract*—In this correspondence, we present a new numerical method for the complex approximation of FIR digital filters. Our objective is to design FIR filters whose absolute error between the designed and desired response is equiripple. The proposed method solves the least-squares problem iteratively. At each iteration, the desired response is updated so as to have an equiripple error. The proposed methods do not require any time-consuming optimization procedure such as the quasi-Newton methods and converge to equiripple solutions quickly. Moreover, by multiplying the arbitrary weighting function on the desired response of the passband and stopband, the errors in the passband and the stopband can be controlled. We show some examples to illustrate the advantages of our proposed methods.**

***Index Terms*—Complex approximation, equiripple design, FIR filters.**

I. INTRODUCTION

In the case of linear-phase FIR filters, since a perfect linear phase can be realized and design algorithms have already been established, it is used in many fields [1]. As is well known, Parks–McClellan algorithm

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